BASIC CONSIDERATIONS ON TEM-MODE HYBRID POWER DIVIDERS

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Abstract

This paper presents a three-way hybrid power divider which splits one signal into three output signals with arbitrary amplitude ratios at all frequencies, and achieves a perfect isolation among all output ports at one frequency.

1. Introduction

The n-way hybrid power divider (henceforth HPD) described by Wilkinson[1] splits an input signal into n equiphase and equiamplitude output signals. Parad and Moynihan[2] presented two-way HPDs with the output signals in phase and with arbitrary amplitude ratios. Cohn[3] presented broad-band two-way HPDs with equiamplitude output signals. Yee, Chang and Audeh[4] presented broad-band n-way HPDs with equiamplitude output signals. Ekinge[5] presented an analysis of broad-band two-way HPDs with arbitrary power-splitting ratios and a transmission line section reactively coupled to an arbitrary degree, in addition to a method of synthesizing two-way HPDs. Tararenko and Goud[6] presented an n-way HPD with arbitrary power splitting ratios only in the appendix of their paper.

The two-way HPD described by Cohn[3] and other papers[2,5] is the simplest one among n-way HPDs. A three-way HPD, which is the next simplest, is the main topic of this paper.

This paper proposes a new three-way HPD, which is made with one section of a three-wire line and with three dummy resistances. It is claimed that the three-way HPD splits an input signal into three output signals with arbitrary amplitude ratios at all frequencies and achieves a perfect isolation among all output ports at one frequency.

The n-way HPD, including a two-way one, can be theoretically designed by utilizing TEM-modes on a lossless transmission multiwire line.[9]

2. Equations of Transmission of a Multiwire Line[7]

It is assumed that all the conductors are perfectly conductive, and that the dielectric medium surrounding the conductors is uniform (with constant ε, μ) and perfectly lossless. Then the propagation of waves on the line can be represented by a real characteristic impedance matrix $[\zeta]$ or a real characteristic admittance matrix $[\eta]$, the transformed complex frequency parameter $\lambda = \tanh \gamma\ell$, and the transmission equations reduce merely to the matrix extension of those for coaxial lines.

We describe only the transmission equation of a three-wire line, because the main purpose of this paper is in analyzing a three-way HPD.

A three-wire line is a (3,3)-port, as shown in Figure 1, and its transmission equations can be expressed as

$$\underset{\sim}{V}_o = c\,\underset{\sim}{V}_\ell + s[\zeta]\underset{\sim}{I}_\ell \tag{1a}$$

$$\underset{\sim}{I}_o = s[\eta]\underset{\sim}{V}_\ell + c\,\underset{\sim}{I}_\ell \tag{1b}$$

where

$$\underset{\sim}{V}_o = \begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \end{bmatrix}, \quad \underset{\sim}{V}_\ell = \begin{bmatrix} V_{1\ell} \\ V_{2\ell} \\ V_{3\ell} \end{bmatrix}, \quad \underset{\sim}{I}_o = \begin{bmatrix} I_{10} \\ I_{20} \\ I_{30} \end{bmatrix}, \quad \underset{\sim}{I}_\ell = \begin{bmatrix} I_{1\ell} \\ I_{2\ell} \\ I_{3\ell} \end{bmatrix}$$

$c = \cos\theta$, $s = j\sin\theta$, $\lambda = j\tan\theta$, $\theta = \beta\ell$

$[\eta]$, $[\zeta]$: nxn real symmetric matrix, $[\zeta] = [\eta]^{-1}$
θ : electrical length of the line section
β : phase constant of the line
ℓ : length of the line.

The characteristic admittance matrix $[\eta]$ is hyperdominant, that is

$$[\eta] = \begin{bmatrix} \eta_{11} & -\eta_{12} & -\eta_{13} \\ -\eta_{12} & \eta_{22} & -\eta_{23} \\ -\eta_{13} & -\eta_{23} & \eta_{33} \end{bmatrix} = \begin{bmatrix} \eta_{1e}+\eta_{12}+\eta_{13} & -\eta_{12} & -\eta_{13} \\ -\eta_{12} & \eta_{2e}+\eta_{12}+\eta_{23} & -\eta_{23} \\ -\eta_{13} & -\eta_{23} & \eta_{3e}+\eta_{13}+\eta_{23} \end{bmatrix} \tag{2}$$

where

$$\eta_{ij} \geq 0 \qquad (i, j = 1, 2, 3)$$

$$\eta_{ie} \geq 0 \qquad (i = 1, 2, 3) \quad .$$

3. TEM-Modes on a Multiwire Line[9]

An n-wire line inside a shielding conductor or over a ground plane supports n independent TEM-modes. TEM-modes on the n-wire line can be represented by voltage vectors and current vectors. We represent a voltage and a current vector of i-th TEM-mode as $\underset{\sim}{U}_i$ and $\underset{\sim}{J}_i$ respectively $(i = 1, \ldots, n)$. The condition that any two of such TEM-modes (i-th and j-th) are independent or orthogonal, is given by[8]

$$\underset{\sim}{U}_i^t\,\underset{\sim}{J}_j \begin{cases} = 0 & (i \neq j) \\ \neq 0 & (i = j) \end{cases} \tag{3}$$

where U_i^t : transpose of U_i.

4. One Section Three-Way Hybrid Power Divider

Starting from an n-wire line (n, n)-port, one can obtain a multiport circuit with less number of ports by open-circuiting (the corresponding currents become zero) or ground-connecting (the corresponding voltages become zero) or short-circuiting (the corresponding voltages become equal) the ports of the original (n, n)-port.

Figure 2 shows three-wire (1, 3)-port circuit with terminal conditions $V_{10} = V_{20} = V_{30}$ for the voltages of a three-wire line shown in Figure 1. The circuit has the following admittance matrix:

$$\begin{bmatrix} I_a \\ -I_{1\ell} \\ -I_{2\ell} \\ -I_{3\ell} \end{bmatrix} = \begin{bmatrix} \frac{\eta_{1e}+\eta_{2e}+\eta_{3e}}{\lambda} & -\frac{\eta_{1e}}{s} & -\frac{\eta_{2e}}{s} & -\frac{\eta_{3e}}{s} \\ -\frac{\eta_{1e}}{s} & \frac{\eta_{11}}{\lambda} & -\frac{\eta_{12}}{\lambda} & -\frac{\eta_{13}}{\lambda} \\ -\frac{\eta_{2e}}{s} & -\frac{\eta_{12}}{\lambda} & \frac{\eta_{22}}{\lambda} & -\frac{\eta_{23}}{\lambda} \\ -\frac{\eta_{3e}}{s} & -\frac{\eta_{13}}{\lambda} & -\frac{\eta_{23}}{\lambda} & \frac{\eta_{33}}{\lambda} \end{bmatrix} \begin{bmatrix} V_a \\ V_{1\ell} \\ V_{2\ell} \\ V_{3\ell} \end{bmatrix} . \tag{4}$$

We take port a as an input port and ports 1ℓ, 2ℓ and 3ℓ as output ports. When one combines conductances G_1, G_2 and G_3 to ports 1ℓ, 2ℓ and 3ℓ respectively, then

$$I_{i\ell} = G_i V_{i\ell} \quad (i=1,2,3) \quad . \tag{5}$$

This three-wire (1,3)-port circuit should satisfy

$$V_{10} = V_{20} = V_{30} \quad . \tag{6}$$

If G_i's (i=1,2,3) satisfy the following equation

$$\frac{\eta_{1e}}{G_1} = \frac{\eta_{2e}}{G_2} = \frac{\eta_{3e}}{G_3} , \tag{7}$$

then

$$V_{1\ell} = V_{2\ell} = V_{3\ell} \tag{8}$$

where

$$\frac{V_{1\ell}}{V_a} = \frac{\eta_{1e}/s}{\eta_{1e}/\lambda + G_1} \quad .$$

In this case, this circuit distributes an input power from port a among ports 1ℓ, 2ℓ and 3ℓ with the power ratios $G_1{:}G_2{:}G_3$, and with equiphase at all frequencies. And the input admittance $(Y_a)_{in}$ looked in from port a is given as:

$$(Y_a)_{in} = \frac{(1+k_2+k_3)(s\,\eta_{1e} + cG_1)\,\eta_{1e}}{c\,\eta_{1e} + sG_1} \tag{9}$$

where

$$\frac{\eta_{2e}}{\eta_{1e}} = k_2 , \quad \frac{\eta_{3e}}{\eta_{1e}} = k_3 \quad .$$

Hence, if the circuit satisfies the following relation

$$\eta_{ie} = G_i \quad (i = 1, 2, 3), \tag{10}$$

then the input admittance is given as

$$(Y_a)_{in} = G_1 + G_2 + G_3 \quad . \tag{11}$$

That is, it can be matched for all frequencies at port a.

Now let's consider the case that one of the output ports becomes an input port. An input port is assumed to be incident on port 1ℓ. In that case, it is desired that port 2ℓ and 3ℓ are decoupled from port 1ℓ. Let's connect conductances $G_1+G_2+G_3$, G_2 and G_3 to ports a, 2ℓ and 3ℓ respectively. Then an input admittance looked in from port 1ℓ can be obtained. But it is never equal to G_1, and port 2ℓ and 3ℓ are never decoupled with port 1ℓ. The fact can be explained by using TEM-modes on the three-wire line. The wave from port 1ℓ travels toward port a. When the wave arrives at the point 10 shown in Figure 2, voltages V_{10}, V_{20} and V_{30} must satisfy Eq. (6).

Here, we should consider that the wave is divided into three TEM-modes and only one of them satisfies Eq. (6). We represent voltage vectors and current vectors for those three TEM-modes as follows:

	Voltage Vector	Current Vector
first mode	$[U_{11} \;\; U_{12} \;\; U_{13}]^t$	$[J_{11} \;\; J_{12} \;\; J_{13}]^t$
second mode	$[U_{21} \;\; U_{22} \;\; U_{23}]^t$	$[J_{21} \;\; J_{22} \;\; J_{23}]^t$
third mode	$[U_{31} \;\; U_{32} \;\; U_{33}]^t$	$[J_{31} \;\; J_{32} \;\; J_{33}]^t$.

Now, let the first TEM-mode satisfy Eq. (6), then

$$U_{11} = U_{12} = U_{13} \quad . \tag{12}$$

From the condition of TEM-modes given by Eq. (3), the current vectors of the second and the third TEM-modes should satisfy

$$J_{i1} + J_{i2} + J_{i3} = 0 \quad (i=2 \text{ and } 3) \quad . \tag{13}$$

In order that the input admittance is equal to G_1 and that port 1 decoupled with port 2ℓ and 3ℓ, only the first TEM-mode should transmit into port a, and the second and the third TEM-modes should be absorbed into some dummy loads. Consequently we consider the circuit which is connected dummy loads G_1', G_2' and G_3' as shown in Figure 3. In order that the three TEM-modes are not disturbed by connecting the dummy loads to the circuit shown in Figure 2, the dummy loads should satisfy the following equation.

$$\frac{G_1'}{G_1} = \frac{G_2'}{G_2} = \frac{G_3'}{G_3} \quad . \tag{14}$$

The first TEM-mode is entirely unrelated to G_i', because the voltages of the first mode satisfies Eq. (8). Therefore, in case that the port a is an input port, G_1', G_2' and G_3' are entirely unrelated for the transmission of a wave into port $i\ell$.

The conductance matrix [G] which is equal to $[\eta]$ given by Eq. (2) can be shown as Figure 4(a). By using the Wye-Delta transformation, the matrix [G] can be transformed into the conductive circuit shown by Figure 4(b). The conductances y_{01}, y_{02}, and y_{03} shown in Figure 4(b) are given as

$$y_{01} = \frac{\Delta}{\eta_{23}} , \quad y_{02} = \frac{\Delta}{\eta_{13}} , \quad y_{03} = \frac{\Delta}{\eta_{12}} \tag{15}$$

where

$$\Delta = \eta_{12}\eta_{13} + \eta_{12}\eta_{23} + \eta_{13}\eta_{23} \; .$$

Let's consider the voltage of point 0 shown in Figure 4(b). If the voltages of three poles 1, 2 and 3 equal each other, then the voltage of point 0 also is equal to the voltage of the three poles. Accordingly the characteristic admittances for the first TEM-mode are given as:

$$\eta_{1e}: \text{for } (U_{11}, J_{11}), \quad \eta_{2e}: \text{for } (U_{12}, J_{12})$$
$$\eta_{3e}: \text{for } (U_{13}, J_{13}) \quad . \tag{16}$$

If the conductances y_{01}, y_{02} and y_{03} satisfy

$$\frac{y_{01}}{\eta_{1e}} = \frac{y_{02}}{\eta_{2e}} = \frac{y_{03}}{\eta_{3e}} \tag{17}$$

or rewritten by using Eq. (15) as:

$$\eta_{1e}\eta_{23} = \eta_{2e}\eta_{13} = \eta_{3e}\eta_{12}, \tag{18}$$

then the voltage of the point 0 is equal to zero. That is, if the elements of $[\eta]$ satisfy Eq. (18), then the characteristic admittances for the second and the third TEM-mode are

$$\eta_{1e} + y_{01}: \quad \text{for } (U_{21}, J_{21}) \text{ and } (U_{31}, J_{31})$$
$$\eta_{2e} + y_{02}: \quad \text{for } (U_{22}, J_{22}) \text{ and } (U_{32}, J_{32})$$
$$\eta_{3e} + y_{03}: \quad \text{for } (U_{23}, J_{23}) \text{ and } (U_{33}, J_{33}) \; . \tag{19}$$

In this case, the three-way HPD shown in Figure 3 can be divided for each TEM-mode and for each port into circuits shown in Figure 5.

Each circuit for the second and the third TEM-mode is a distributed-parameter high pass circuit. Therefore, if $G_i' = G_i$ (i = 1, 2, 3), then the six circuits for the second and the third TEM-modes can be matched at the electrical length of 90^o. That is, port 1ℓ can be matched and port 2ℓ and 3ℓ are decoupled with port 1ℓ at that frequency.

This result of three-way HPD can be easily extended to n-way HPD. That is, if the following equations satisfy,

$$G_i = G_i' , \text{ and } G_i/Y_{oi} = \text{const.} \quad (i = 1, \ldots, n) \tag{20}$$

where

G_i : conductance for output
G_i' : conductance for dummy load
Y_{oi} : characteristic admittance for distributed line,

then n-way HPD which have splitting-power ratios $G_1{:}G_2{:}\ldots{:}G_n$ at one frequency, can be obtained.

Conclusion

A basic method of analyzing a three-way hybrid power divider has been presented. A new three-way power divider which is made with one section of three-wire line and with three dummy resistances, is proposed. And it is clarified that the three-way one splits an input signal into three output signals with arbitrary amplitude ratios at all frequencies and achieves a perfect isolation among all output ports at one frequency.

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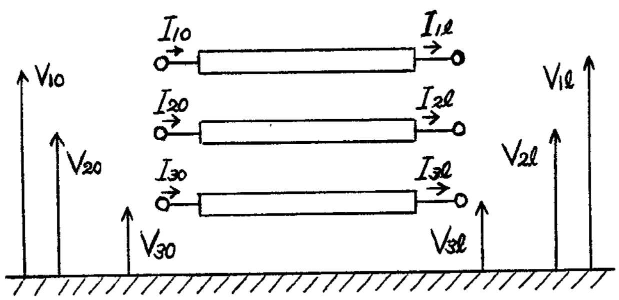

FIG. 1. THREE-WIRE LINE.

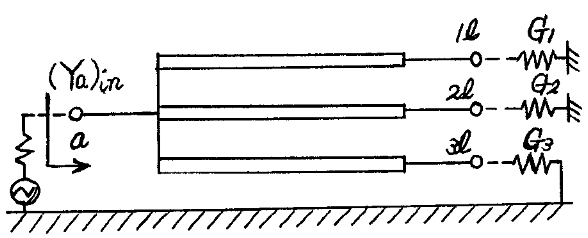

FIG. 2. THREE-WIRE (1, 3)-PORT CIRCUIT.

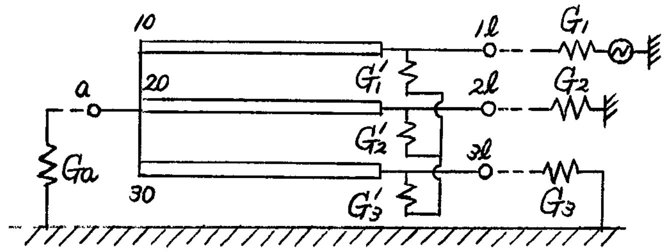

FIG. 3. THREE-WAY HYBRID POWER DIVIDER.

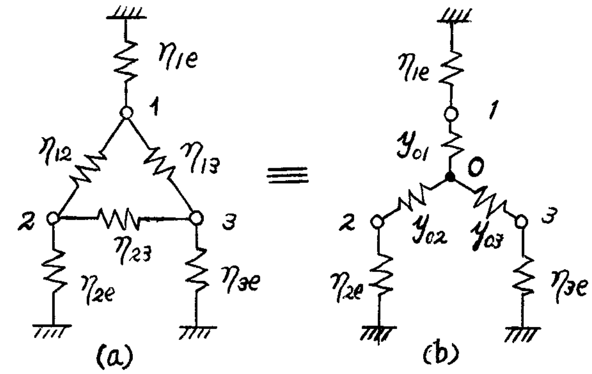

FIG. 4. WYE-DELTA TRANSFORMATION OF CONDUCTIVE MATRIX [G].

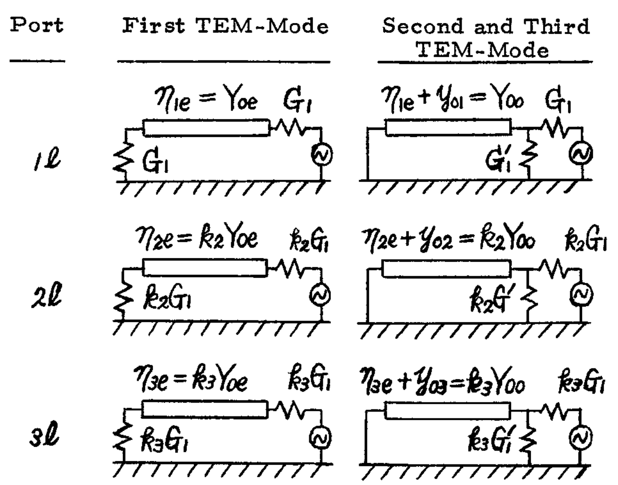

FIG. 5. DIVIDED EQUIVALENT CIRCUITS FOR EACH TEM-MODE AND EACH PORT.